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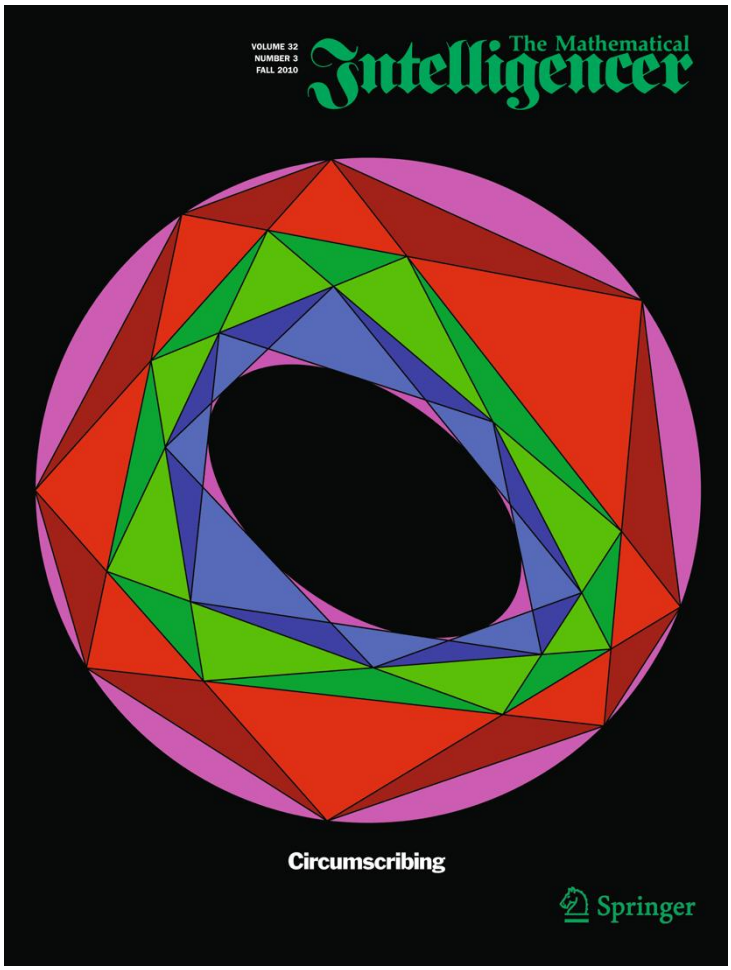

A Mathematical Trip to Princeton

Ezra Brown

Does your hometown have any mathematical tourist attractions such as statues, plaques, graves, the café where the famous conjecture was made, the desk where the famous initials are scratched, birthplaces, houses, or memorials? Have you encountered a mathematical sight on your travels? If so, we invite you to submit an essay to this column. Be sure to include a picture, a description of its mathematical significance, and either a map or directions so that others may follow in your tracks.

The phone rings and a familiar voice speaks. "Since you're going to Princeton this summer, you'll surely be taking a tour of the Princeton campus, and their math department. When you go there, be sure to take note of a sculpture near the Math Department Commons Room. This piece involves three mathematicians: One was the artist, another commissioned the piece, and the third was the one to whom it was dedicated. When you get back, tell me what you ... what? No time? I'm sorry, you *make* the time."

Orders are orders, so you make the time and tour the beautiful Princeton University Campus. There, you find that sculpture, along with an ivory tower and many other sights worth seeing.

The Princeton University mathematics department may or may not be in first place in the mythical USA Mathematics Marathon, but it is certainly up there in the lead pack. Two Fine Halls have housed the Princeton math department, both named for Henry Burchard Fine (1858–1928), the first chair of mathematics at Princeton and President of the American Mathematical Society in 1911–1912. The first building was opened in 1930; it was renamed Jones Hall in 1968 when the second and current Fine Hall was built.

In the immediate vicinity of the current Fine Hall are one building and two sculptures worth a second look. The building is the Lewis Science Library [4], which opened in 2008. Designed by architect Frank O. Gehry, it is a mathematical marvel in its own right. (Go inside and look around: It's time well spent.) Viewed from the Fine Hall tower, it resembles a cell complex (Figure 1).

The library is one among many architectural gems at Princeton, as an architect friend from high-school and his historian wife demonstrated during a campus tour. The library stands next to Fine Hall on what was formerly the site of an informal volleyball court used by the math faculty and students. Apparently, when Fields Medalist William Thurston was on the faculty, he supplied the net, which he put up in the morning and took down in the evening.

Two steel sculptures of mathematical character stand in the vicinity. The older one is Alexander Calder's 26-foot sculpture "Five Disks: One Empty;" [3] dedicated in 1971, it stands in the Fine Hall courtyard plaza between the mathematics and physics departments. (Maybe it looks like a horse, and maybe it doesn't–Modern Art is like that.) The newer one is Richard Serra's "The Hedgehog and the Fox," [5] a trio of 90-foot long and 15-foot high nested serpentine steel ribbons standing just east of the library. It's fun to walk through this huge installation and listen to the echoes. Appropriately, when viewed from the Fine Hall tower, "The Hedgehog and the Fox" looks like a triple integral sign (Figure 2).

Figure 1. The Gehry Library (photo by Sharon Sells).

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Figure 2. The Serra Sculpture (photo by Sharon Sells).

The present Fine Hall is an imposing 12-story tower—the tallest building on the Princeton campus. The upper nine floors seem identical, with two elevator doors, two stairwell doors, a dozen or so offices and a seminar room. Each floor's office directory is on a wall near the elevators, and mathematical tourists will recognize some famous names. All the doors are closed. Apparently, the present Fine Hall was designed by an architect who did not have the collaborative/social nature of mathematical research in mind. The result was the quintessential Ivory Tower, an appropriate name because, as one recent Ph.D. graduate from the department put it, the corridors are 99 44/100% empty.

The sameness of those upper floors led, according to one source, to some mischief. The night before April Fools Day one year, some of the students rewired the elevator buttons so that, for example, pressing the button for the fifth floor might bring the elevator to the eighth floor. The numbers on had been similarly switched accordingly. The next morning, all the professors came in, rode the elevators to "their" floors, went to "their" offices–but nobody's keys worked. (Allow yourself a wry chuckle at the bedlam that ensued.) On the matter of who did this, the source was as silent as the corridors of those upper nine floors.

More important than the appearance of the building, however, are the results of the mathematical training occurring therein. These results have been astonishing, and they are reflected in the group photographs of each entering class of graduate students from 1966 to the present that line the third-floor corridor. You can watch changes unfold as you go through the years. The first class was all male and they wore suits (that didn't last long); eventually, women appeared in the pictures. Among the pictures you see many eminent mathematicians, including colleagues from your home institution and other workplaces, three Fields Medalists, numerous Putnam Fellows and International Math Olympiad participants, a MacArthur Fellow, and a coauthor—all looking very youthful. It's quite an impressive showcase. It is true that four Princeton Ph.D. mathematicians have been Fields Medalists, not three. However, the student days of John Milnor, the first Princeton Ph.D. to receive a Fields Medal, predate the series of pictures.

The next thing you see is in the middle of the run of group photographs, namely, the door of Room 310. Unlike practically every other door in the building, 310 is completely covered with signs, figures and pictures in a fascinating crazy quilt, reflecting the interests of the office's inhabitant, John H. Conway. There's no official name on the door, but a sign advertising "Conway: $9.99" gives it away.

Then you see the mathematical sculpture, entitled "The Third Constant of Euclidean Geometry," and it is very striking. This polished five-foot high tower, beautifully made of Inner Mongolian black granite, is the work of the eminent sculptor-mathematician Helaman Ferguson (Figures. 3, 4).

The piece stands in a prominent spot across from the departmental Commons Room, back-lit by light from a south-facing window. Ferguson's work honors the memory and the mathematics of the late Princeton Professor Fred Almgren, who worked in geometric measure theory. It was commissioned by Professor Jean Taylor, Almgren's second wife and his first Ph.D. student, and dedicated in 2000. This sculpture vividly captures Professor Almgren's startling and comprehensive theorems on generalizations of the isoperimetric inequality and mass-minimizing hypersurfaces, announced in [1] and described in detail in a legendary 1,700-page preprint. This latter work was edited by Jean Taylor and Vladimir Scheffer and published in a single volume in 2000 [2]—truly a labor of love.

The constant of the sculpture's title is called γ, defined by

$$\gamma(n+1) := ((n+1)/2)!^{1/n}/((n+1)\sqrt{\pi})^{(n+1)/n}.$$

For $2 \le n \le 8$, $\gamma(n)$ is written out in base 10 and base k, $2 \le k \le n$ on each of seven levels of the sculpture. Each level is a regular n-gon, and the sculpture flows beautifully between levels. Appropriately, for the sculpture's close connection with π, the nearest room is Room 314.

AUTHOR

EZRA (BUD) BROWN grew up in New Orleans, has degrees from Rice and LSU, and has been at Virginia Tech since 1969, where he is currently Alumni Distinguished Professor of Mathematics. Most of his research has been in number theory and combinatorics, but one of his favorite papers was written with a sociologist. During the summers, he does applied mathematics in the Washington, DC Area. He has received some writing awards from the Mathematical Association of America. He enjoys singing in operas, playing jazz piano, gardening, and kayaking. He occasionally bakes biscuits for his students.

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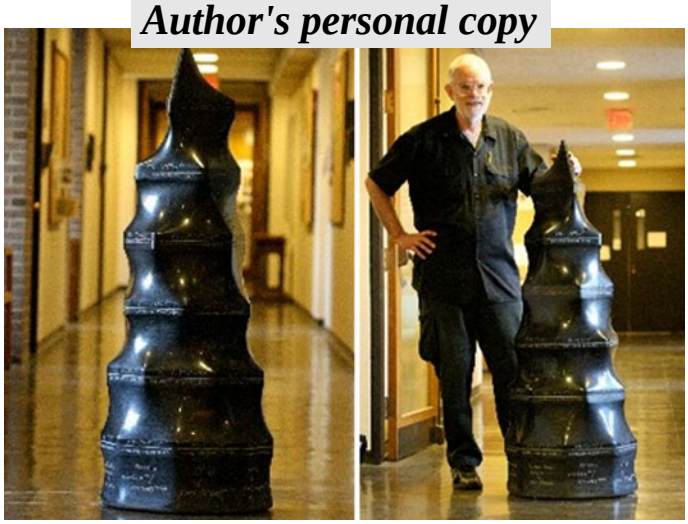

Figure 3. "The Third Constant" and its creator (photos by Georg Glauser).

Figure 4. "The Third Constant" (detailed view; photo by Georg Glauser).

Finally, Helaman Ferguson tells a story about the sculpture's installation. Inner Mongolian black granite is heavy, and those weight limits printed on elevators are not just for show. Ferguson calculated that the combined weight of his assistant, himself, the sculpture and the equipment needed to move and install the sculpture was *just barely* under the capacity of the freight elevator. On an agreed-upon date, the two of them unloaded the piece from Ferguson's truck and carefully moved sculpture and equipment into the freight elevator. Just then, someone who knew about the move ran around the corner and leaped joyfully into the elevator just as the doors were closing ... thus loading the elevator beyond its capacity.

Now, how can you ask the latecomer to leave, when the latecomer is John Conway? Well, you can't; you press the button and hold your breath. The elevator shuddered a bit and rose uncertainly to the third floor. After that bit of excitement, the rest of the installation was routine.

What else of mathematical interest is there to see in Princeton? Merely the Institute for Advanced Study—but that's another story!

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[1] Frederick J. Almgren Jr, Optimal Isoperimetric Inequalities, Bull. Amer. Math. Soc. (N.S.), **13**, #2 (October 1985), 123–126.

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[3] http://blogs.princeton.edu/aspire/2009/01/sculpture_at_princeton_the_putnam_collection.html

[4] http://www.princeton.edu/main/news/archive/S20/84/49I22/index.xml?section=featured

[5] http://www.lera.com/projects/usnj/serraprinceton.htm