> "Many of history's famous word problems first appeared in my book: mixtures, percentage, discount, the wolf-goat-cabbage, the hen-and-a-half laying an egg-and-a-half in a day-and-a-half, etc."

A Conversation with Leonardo Pisano

Ezra Brown
Virginia Tech

Three in the morning, too much coffee, and hours of poring over the new English translation of Fibonacci's 1202 masterwork *Liber Abaci*—that could be the only explanation for why the great man himself materialized in the only vacant chair in the room. After I recovered my composure, we had this conversation:

Math Horizons: Are you the illustrious Fibonacci?

Leonardo Pisano: You talkin' to me?

MH: Well, yes. . .at least, I thought you were . . .

LP: . . .the thirteenth century mathematician Leonardo Pisano? Yes, I am. . .or rather, I was. . .time gets muddled after a few centuries, you know.

MH: But you are now universally known as Fibonacci, aren't you?

LP: Look, Pal, that tag was laid on me by this nineteenth century historian Guillaume Libri. Nobody called me Fibonacci in my lifetime. In my time, not everyone had last names. My name meant "Leonardo of Pisa," my father was Guilielmo and "Bonaccio" is a family name. But hey, forgive and forget. At least, they haven't forgotten me.

MH: Yes, well, Leonardo, mathematicians and students of our time are really curious about your life and your work, so may I ask you some questions?

LP: Go right ahead, Pal. What do you want to know?

MH: First of all, when were you born?

LP: Born? Who knows. It was considered bad luck to write the name of a child until you reached a certain age. Record keeping was spotty even in the best of times, and Pal, those were the best of times for Pisa. They began building the bell tower in the 1170s, but stopped after putting up only three stories when they realized what rotten ground they'd chosen for the site. Anyway, I always remember at least part of the tower being there. According to the preface of *Liber Abaci*, Pop sent word for me to come to the North African seacoast port of Bugia, sometime before I was eighteen. That was in 1192, so let's just say that I was born in the late 1170s.

MH: What was it like growing up in Pisa?

LP: Pisa, Genoa, Amalfi, and Venice were the Republiche Marinare, the four Maritime Republics. It was a trade alliance; they had divided up the trade routes to the Mediterranean countries and beyond. So, Pisa was a pretty lively place, with the pepper, leather, and fur trades and so many people coming and going. . .a great place to grow up in. Rough, at times, and filthy-dirty by your standards, but a great place. Pisa never was very big, by the way; its population in my time was maybe 10,000. We had no king or prince who ruled over us; we were a republic. You know what a republic is, don't you, Pal?

MH: Certainly.

LP: Well, then, because times were good, the merchants pretty much ruled the roost. They put kids to work early in those days; labor was scarce, and children were needed in the shops. My family figured out that I wasn't just an ordinary kid, but one with a head on his shoulders, so I was taught to make change and read a ledger. My family was big on literacy. You'll never guess who taught me to read.

MH: Who was it?

LP: My grandmother. She had been taught to read by the son of a family friend. The son had been to the University of Bologna, was passing through Pisa, and stayed with Great Grandpa for a while. He was quite taken with Grandma, who was a small child, and would patiently answer her questions about the world. He had a book of poems, and would read to her, and eventually, she figured out the trick of reading and began to follow along. Fifteen years later, the student came back and married her. He's my grandpa. Anyway, she taught me to read out of that same book of poetry.

MH: Tell us about your education.

LP: We began religious training as six-year-olds, but as I said, even before that I learned to read. Hanging around merchants, peddlers, money-lenders, etc. I learned your basic "street math." For instance, if you were working in a shop and an

important buyer came in from Barcelona, he might not have Pisan money. You had to know rates of exchange and convert between many different currencies. I put quite a few problems involving foreign exchange in *Liber Abaci*, you know. Merchants had to work quickly and accurately, juggling money from Amalfi, Byzantium, Rome, Venice, etc. In Bugia, I was introduced to the wonders of Indian figures, including the zephir, and eventually the al-jabr. My father hired a skilled teacher to instruct the Pisan merchants in the Indian figures and I was included in that instruction. I think it was Pop's plan for me to be a merchant, too. But the haggling and worrying about making sales and fretting about prices of goods and dealing with stubborn customers—that wasn't for me. So I became a teacher and a writer.

MH: How did that happen, and how did you come to write the *Liber Abaci*?

LP: The reasons are right there in the prologue, but I guess what you really want to know is, why me? I was the right man for the job—that's why. You see, I learned quickly and could explain the use of Indian figures in a way that merchants and other people could understand. There was clearly a need for good instruction, and when I returned to Pisa after traveling the Mediterranean world, it seemed like a good idea to write a book on the Indian figures and the al-jabr, complete with many examples that merchants would understand. Many of history's famous word problems are in there in one form or another: mixtures, percentage, discount, the wolf-goat-cabbage, the chicken-and-a-half laying an egg-and-a-half in a day-and-a-half, etc. The man-who-traveled-in-twelve-cities problem is an example of exponential decay, 450 years before Newton's Law of Cooling. I included problems with large numbers, too, so that people could see the power of Indian figures. By the way, the largest number in the book is 3^{64}. I also included proofs in the Euclidean manner, so that learned scholars would believe what I wrote. As long as I was introducing Indian figures to the merchants of Pisa and the Maritime Republics, it would have been even better to come up with some powerful new notation. But eight hundred years of hindsight, and all that—anyway, I wrote it, and revised it in 1228. People now say that most of it wasn't original, and they're right, but so what? That wasn't why I wrote it. It was a big hit in my time. Of course, the Firenzi, the Florentine merchants objected to using the Indian figures, claiming that they invited fraud and forgery. What people forget, Pal, is that Florence and Pisa were never on the best of terms, and for them to use a method suggested by a Pisano…well, I wasn't surprised that they passed laws against the use of Indian figures, laws that lasted until the fifteenth century, when printing with movable type came along.

MH: You wrote a *Book of Squares*, so why not a *Book of Cubes*?

LP: Why didn't I write a *Liber Cuborum*? Hey, Pal, that's an easy one. I've got a reputation to maintain, and I didn't consider myself an expert...besides, it was gonna be hard enough for people to read *Liber Quadratorum*, even though squares are really not that tough. One more thing: in those days, I didn't believe that you could solve cubic equations using al-jabr...300 years later, a bunch of Giovannis-come-lately proved me wrong—and they were all from my part of the world.

MH: Did you leave any unsolved problems that you'd like to return to in the modern day?

LP: Sure, but most of them got solved in the meantime. The sixteenth century Italian lads got the cubic (and quartic) formulas, Newton extended and generalized my approximations, and Fermat and Euler went through my number theory like Alexander the Great. In your time, a fellow named Tunnell solved the congruous number problem, recursion has been taken over by the wizards with those magic machines, and iteration has literally turned to Chaos! What's left for a kid from Pisa? Precious little.

MH: What do you think about the vast amount of research that has been done on your Fibonacci numbers and their generalizations?

LP: I like it! It's very gratifying to see your name everywhere, Pal; there's even an entire society devoted to the Rabbit Numbers. But the most startling wrinkle on the original problem happened in the last five years. It's Divakar Viswanath's amazing work on the sequence defined by $t_1 = t_2 = 1$ and for $n > 2$, $t_n = \pm t_{n-1} \pm t_{n-2}$, where the signs are independent and either + or – with probability 0.5. He proved that, with probability 1, $\lim_{n\to\infty} \sqrt[n]{|t_n|} = 1.13198824\ldots$ Some people are calling this number a new constant that rivals the Golden Ratio $\varphi = \left(1+\sqrt{5}\right)/2$, the limit of the ratio of consecutive Fibonacci numbers. Hey, I'm impressed!

Cartoon by John Johnson

MH: Did you consider yourself a "pure" or an "applied" mathematician, or did you never even consider such a distinction?

LP: Let's put it this way. I wrote *Liber Abaci* as a practical text for merchants and as a way to spread the word about the Indian figures and al-jabr. I wrote *Liber Quadratorum* for my own benefit. I wrote up *Flos* as a record of the Imperial contest, so nobody'd forget. You figure it out, Pal.

MH: Does it surprise you that modern mathematicians consider algebra and numerical analysis to be largely separate subjects?

LP: Not really. Mathematics has grown so large that you have to split things up. Nobody since Hilbert and Poincaré has come close to understanding it all, or even most of it.

MH: Did you anticipate that the solution of the quintic would present so much difficulty?

LP: Actually, I never thought anyone would solve the cubic, let alone the quintic.

MH: Why would a king be interested in meeting a mathematician?

LP: The Holy Roman Emperor Frederick II was a complex man. He could be ruthless and cruel, but he was also interested in art, music, and the intellect—a patron of the arts, you might say. When he traveled to Pisa, his advisor Giovanni di Palermo mentioned that Leonardo Pisano was someone worth meeting, so they arranged for me to be presented at Frederick's court.

MH: In your time, were there any areas of mathematics in which Europe excelled, but Indian, Arabic, or other Eastern mathematicians lacked?

LP: Not one single area, Pal. There wasn't anything there to build on. The Crusades brought Europe in contact with the much more advanced cultures of the East, and scholars had begun some impressive translation projects, but the only reason I knew about Euclid was that there were Arabic translations. Here's what the eight centuries of European mathematics after Pappus and before me produced: zilch.

MH: Which of your many achievements do you consider your best work?

LP: You don't ask easy questions, Pal. I like the proof in *Liber Quadratorum* of the formula for $1^2+2^2+3^2+\cdots+n^2$ that uses what you now call telescoping sums. As I said before, the Rabbit Problem became a whole lot bigger than just a simple counting problem, because recursion and iteration are hot topics in your time. In Chapter 2 of *Liber Abaci*, I mentioned Casting Out Thirteens and other checks on calculations; before my time, it was only Casting Out Nines. This was modular arithmetic 600 years before Gauss' *Disquisitiones Arithmeticae*; this was a detailed error-detecting scheme 750 years before Hamming's parity checks and error-correcting codes. I like what I did with the congruous numbers problem, finding a rational number x such that x^2+5 and x^2-5 are both squares, which Euler extended way beyond what I did. The first approximation to the cube root of a noncube had been around quite a while—the Chinese had it, I think—but I was the first to spell out the second approximation. It's the next step in Newton's method, but in my day, who knew about Newton? Then there's that six-sexagesimal-place approximation to the solution of $x^3 + 2x^2 + 10x = 20$ that I showed to Frederick II. . .the looks of amazement on his face and on Giovanni di Palermo's ugly mug are still with me after over 750 years!

MH: Finally, do you have any words of wisdom for our students?

LP: That's easy: join the MAA and subscribe to *Math Horizons*! Listen, Pal, I'm late for a chat with Leonardo da Vinci, Leonhard Euler, and Leonard Carlitz. Around those three, I'm always in the dumbest quarter of the room, so arrivederci, Pal!

Then he disappeared. The next morning, my wife asked me, "I heard voices downstairs last night; who were you talking with?" Well, what would you say?

Suggested Reading

Start with the Master and read L. E. Sigler's marvelous English translations, Fibonacci's *Liber Abaci*, Springer-Verlag, New York, 2002, and "The Book of Squares (English Translation of *Liber Quadratorum*)," Academic Press, New York, 1987. The references in these books are rich in information about Leonardo and his work. The website `http://www.maa.org/devlin/devlin_3_99.html` is a readable account of Divakar Viswanath's work.