

Whodunit? Fifteen Puzzlers for April First

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“Things are seldom what they seem: Skim milk masquerades as cream...”

–Sir Arthur Sullivan

“Clio, the muse of history, often is fickle in the matter of attaching names to theorems!”

–Carl B. Boyer

The Quiz

1. L'Hospital's Rule is as follows. If f and g are differentiable on some interval I about a , except possibly at a , with $g' \neq 0$ on $I - \{a\}$, and if $\lim_{x \rightarrow a} f(x) = \lim_{x \rightarrow a} g(x) = 0$, or if these limits are both infinite, then

$$\lim_{x \rightarrow a} \frac{f(x)}{g(x)} = \frac{\lim_{x \rightarrow a} f'(x)}{\lim_{x \rightarrow a} g'(x)},$$

provided the latter limit exists or is infinite. L'Hospital's Rule is due to which of these? (a) Johann Bernoulli (b) Jacob Bernoulli (c) Julie Bernoulli (d) Guillaume François Marquis de l'Hospital (e) Gottfried Wilhelm Leibniz.

2. Laplace's Equation $\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2} = 0$ due to (a) Alexis Clairaut (b) Leonhard Euler (c) Jean le Rond D'Alembert (d) Joseph Louis Lagrange (e) Pierre Simon de Laplace.

3. The Gaussian or normal distribution is the probability density function defined by

$$f(x) = \frac{1}{\sqrt{2\pi}\sigma} e^{-(x-\mu)^2/2\sigma^2},$$

where μ and σ denote the mean and variance, respectively. The Gaussian distribution is due to (a) Karl Friedrich Gauss (b) Abraham deMoivre (c) Brooke Taylor (d) Pierre Simon de Laplace (e) Adrien-Marie Legendre.

4. Pascal's Triangle is the triangular arrangement of positive integers in which the k th entry in the n th row is equal to $C(n, k)$, the number of k -combinations of n objects. Pascal's Triangle is due to (a) Blaise Pascal (b) Michel Stifel (c) Niccolo Tartaglia (d) Yang Hui (e) Zhu Shijie.

5. Simpson's Rule for numerical integration states that if f is continuous on the interval $[a, b]$, if n is an even integer and if $\{x_0, x_1, \dots, x_n\}$ is the regular partition of $[a, b]$ into n equal subintervals, then

$$\int_a^b f(x) dx \approx \frac{b-a}{3n} (f(x_0) + 4f(x_1) + 2f(x_2) + \dots + 2f(x_{n-2}) + 4f(x_{n-1}) + f(x_n)).$$

Simpson's Rule is due to (a) Homer Simpson (b) the Moscow Papyrus (c) James Gregory (d) Isaac Newton (e) Thomas Simpson.

6. Pell's Equation $x^2 - dy^2 = 1$ is due to (a) Bhaskara (b) Brahmagupta (c) Lord Brouncker (d) Pierre Fermat (e) John Pell.

7. A Vandermonde matrix is the following $n \times n$ matrix V_n formed from a set $\{a_1, \dots, a_n\}$:

$$V_n = \begin{pmatrix} 1 & x_1 & x_1^2 & \cdots & x_1^{n-1} \\ 1 & x_2 & x_2^2 & \cdots & x_2^{n-1} \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 1 & x_n & x_n^2 & \cdots & x_n^{n-1} \end{pmatrix}$$

The Vandermonde Determinant Theorem states that $\det(V_n) = \prod_{1 \leq i < j \leq n} (x_j - x_i)$, and is due to (a) Augustin-Louis Cauchy (b) Arthur Cayley (c) Joseph Louis Lagrange (d) Gottfried Wilhelm Leibniz (e) Alexandre-Theophile Vandermonde.

8. The Archimedean Principle states that for any real number a , there exists a positive integer n such that $n > a$. The Archimedean Principle is due to (a) Archimedes (b) Euclid (c) Eudoxus (d) Richard Dedekind (e) Archie Bunker.

9. The Sylow Theorems are three major results about the structure of finite groups, and are as follows: let G be a finite group of order $p^k m$, where p is a prime and p and m are relatively prime. Then (1) G has a subgroup of order p^k , called a p -Sylow subgroup, (2) all p -Sylow subgroups of G are conjugate, and (3) the number of p -Sylow subgroups divides the order of G and is of the form $np + 1$ for some integer $n \geq 0$. The Sylow Theorems are due to (a) Peter Ludwig Mejdell Sylow (b) Georg Frobenius (c) Arthur Cayley (d) Augustin-Louis Cauchy (e) Sophus Lie.

10. The equation $e^{i\theta} = \cos(\theta) + i \sin(\theta)$, known as Euler's Formula, is due to (a) Leonhard Euler (b) Roger Cotes (c) Abraham deMoivre (d) James Bernoulli (e) Colin Maclaurin.

11. The Catalan numbers $C_n = \frac{1}{n+1} \binom{2n}{n}$ have many combinatorial interpretations, such as the number of distinct ways to parenthesize a string of n symbols. The first few are 1, 2, 5, 14, 42, \dots . The Catalan numbers are due to (a) Eugène Charles Catalan in 1838 (b) Euler (c) Thomas Kirkman (d) Antu Ming (e) Johann Andreas von Segner.

12. The sequence 1, 1, 2, 3, 5, 8, 13, 21, 34, \dots is commonly known as the Fibonacci sequence, and was first used by (a) Fibonacci (b) Leonardo of Pisa (c) Acarya Hemachandra (d) Bugs Bunny or some other wascawwy wabbit (e) Édouard Lucas.

13. Stirling's Approximation $n! \approx (n/e)^n \sqrt{2n\pi}$ for $n!$ is due to (a) Gabriel Cramer (b) Abraham de Moivre (c) Gottfried Wilhelm Leibniz (d) Colin Maclaurin (e) James Stirling (f) Brook Taylor.

14. The Maclaurin expansion of a function f having derivatives of all orders at 0 is given by $f(x) = f(0) + \sum_{n=1}^{\infty} f^{(n)}(0) \frac{x^n}{n!}$ and is due to (a) Gabriel Cramer (b) Abraham de Moivre (c) Gottfried Wilhelm Leibniz (d) Colin Maclaurin (e) James Stirling (f) Brook Taylor.

15. Cramer's Rule for linear systems is as follows. If the $n \times n$ matrix $M = [a_{ij}]_{n \times n}$ has a nonzero determinant, then the system $\sum_{j=1}^n a_{ij} x_j = b_i$ ($i = 1, \dots, n$) of n linear equations in n unknowns has a unique solution given by

$$x_j = \frac{\det M_j}{\det M},$$

where M_j is the matrix obtained from M by replacing the j th column by the column vector $(b_1, \dots, b_n)^T$. Cramer's Rule is due to (a) Gabriel Cramer (b) Abraham de Moivre (c) Gottfried Wilhelm Leibniz (d) Colin Maclaurin (e) James Stirling (f) Brook Taylor.

Bonus Question. Boyer’s Law states that mathematical formulas and theorems are usually not named after their original discoverers. Boyer’s Law is due to (a) Carl B. Boyer (b) someone else.

The Answers

1. (a) 2. (b) or (c) 3. (b) 4. (d) 5. (b) 6. (c) 7. (a) 8. (c) 9. (b) 10. (b) 11. (d) 12. (c) 13. (b) 14. (e) or (f) 15. (a) Bonus question. (b)

The Explanations

1. (a) Johann Bernoulli. The rule appeared in L’Hospital’s 1696 book “Analyse des infiniment petits pour l’intelligence des lignes courbes”. It is now known that the result was due to Johann Bernoulli in the early 1690’s. Much ink has been spilled about both the origin and the (mis)application of L’Hospital’s Rule.

2. (b) Euler or (c) D’Alembert. Their 1761 papers on hydrodynamics introduced the equation to the mathematical world, but Laplace’s work on potential (1785) and his monumental *Mécanique Céleste* (1799) made it widely known.

3. (b) Abraham de Moivre. The normal distribution was first introduced by Abraham de Moivre in an article in 1734 (reprinted in the second edition of his *The Doctrine of Chances*, 1738) in the context of approximating certain binomial distributions for large n . His result was extended by Laplace in his book *Analytical Theory of Probabilities* (1812), and is now called the theorem of de Moivre-Laplace. Laplace used the normal distribution in the analysis of errors of experiments. The important method of least squares was introduced by Legendre in 1805. Gauss, who claimed to have used the method since 1794, justified it rigorously in 1809 by assuming a normal distribution of the errors.

4. (d) Yang Hui. Yang (ca. 1275) displayed the triangle in one of his works, according to Zhu Shijie in his own 1303 treatise “The Precious Mirror of the Four Elements”. Nicolo Tartaglia first published the generalization of the figurate numbers in 1523 and a tabular form of the triangle appears 30 years later in his *General Treatise*. Stifel displayed the left half of the triangle, through the sixteenth row, in a 1544 treatise. Pascal’s principal work on the triangle dates from 1665 and was used in connection with his work on probability.

5. (c) James Gregory. The Moscow Papyrus (ca. 1890 BCE) contains the formula $V = \frac{h}{3}(a^2 + ab + b^2)$ for the volume of a truncated pyramid of height h and base widths a and b . Let $A(x)$ be the area of a cross-section parallel to the bases at the height x . Then $A(0) = b^2$, $A(h) = a^2$, and in general $A(x) = (b + \frac{a-b}{h}x)^2$. Thus, $V = \int_0^h A(x) dx$. Simpson’s Rule for $n = 2$ is exact for quadratic polynomials and gives the volume as $V = \frac{h}{6}(A(0) + 4A(h/2) + A(h))$, and a little algebra shows that this agrees with the result from the Moscow Papyrus. This degree of mathematical sophistication is perhaps unique in ancient Egyptian mathematics. The earliest recognizable forms of Simpson’s Rule, however, appear in works by Gregory (ca. 1668) and Newton (1711); these precede Simpson’s 1743 publication of the formula that bears his name. As for Homer Simpson, well, “D’oh!” Ironically, Thomas Simpson attributed the Rule to Newton!

6. (b) Brahmagupta. In his *Brahmasphutasiddhanta* from 628, Brahmagupta described some methods for solving the equation $x^2 - dy^2 = 1$ for several values of d . He gives $x = 1766319049, y = 226153980$ as the smallest solution for $d = 61$, an impressive piece of hand calculation. Bhaskara considerably extended Brahmagupta's work in 1150. In 1658, William Lord Brouncker (1620-1684) discovered a general method of solution equivalent to the continued fraction algorithm, in response to one of Fermat's challenge problems. John Pell's name was attached to the equation by Euler in a classic case of misattribution: Pell had nothing to do with it. The website <http://www.gap-system.org/history/HistTopics/Pell.html> tells the story very well.
7. (a) Augustin-Louis Cauchy. Vandermonde studied this matrix in the 3×3 case in the 1770's, but the theorem for general $n \times n$ matrices is due to Cauchy in an 1815 paper.
8. (c) Eudoxus. Archimedes (late 3rd century BCE) used this principle in several treatises, including *On the Sphere and Cylinder* and *On the Quadrature of the Parabola*, and he attributes the principle to Eudoxus (early 4th century BCE). A form of the principle appears in Book V of Euclid's *Elements* (early 3rd century BCE). Dedekind's Completeness Axiom is late 19th century. Archie Bunker knows all about Principals ("I was always gettin' sent to the Principal's office"); his only Principle is "Keep It All In The Family."
9. (b) Georg Frobenius. Frobenius proved Sylow's theorems for arbitrary groups in 1884. Sylow's original proofs from 1872 are for permutation groups. Cauchy's theorem on the existence of an element of order p in a finite group of order divisible by p was motivation for Sylow's work. Cayley and Lie had nothing to do with it – they are just two more giant groupers.
10. (b) Roger Cotes. The equivalent equation $\ln(\cos \theta + i \sin \theta) = i\theta$ was discovered by Cotes about 1713. De Moivre's related formula $(\cos \theta + i \sin \theta)^n = \cos n\theta + i \sin n\theta$ appeared in 1722, and Euler used it to derive the result we now call Euler's formula. The celebrated equation $e^{i\pi} + 1 = 0$ is due to Euler and first appeared in 1748 in *Introductio in analysin infinitorum*.
11. (d) Antu Ming. The Mongolian mathematician and astronomer Antu Ming (ca. 1692 - 1763) wrote about these numbers around 1730. Segner's 1758 recurrence formula gives the solution to Euler's polygon division problem $E_n = E_2 E_{n-1} + E_3 E_{n-2} + \dots + E_{n-1} E_2$ (J. A. von Segner, *Enumeratio modorum, quibus figurae planae rectilineae per diagonales dividuntur in triangula*, *Novi Comm. Acad. Scient. Imper. Petropolitanae*, 7 (1758/1759), 203-209). In 1838, Catalan showed that the number of ways of parenthesizing a product of $n + 1$ symbols using n pairs of parentheses is equal to the n th Catalan number C_n .
12. (c) Acarya Hemachandra. Hemachandra presented what is now called the Fibonacci sequence around 1150, about 50 years before Fibonacci (1202). He was counting the number of rhythmic patterns $p(n)$ made up of single-beat and double-beat notes of length n and showed that these could be formed by adding a single-beat note to a pattern of length $n - 1$ or a double-beat note to one of length $n - 2$. The resulting recurrence $p(n) = p(n - 1) + p(n - 2)$ with the initial conditions $p(0) = p(1) = 1$ define the Fibonacci sequence.) The famous Rabbit Problem of Leonardo of Pisa, also known as Fibonacci, appeared as Chapter 12, Part 7, Problem 18 in the *Liber Abaci* from 1202 and introduced the sequence to the European world. Lucas gave the sequence its name in the 19th century. The Fibonacci sequence is the only bit of mathematics that Bugs Bunny knows.
13. (b) Abraham de Moivre. The approximation $n! \approx cn^{n+1/2}e^{-n}$ appears in de Moivre's 1730

work *Miscellanea Analytica*. Stirling's own subsequent contribution was the exact value of $\sqrt{2\pi}$ for the constant c .

14. (e) James Stirling or (f) Brook Taylor. Taylor described the series expansions

$$f(x) = f(a) + \sum_{n=1}^{\infty} f^{(n)}(a) \frac{(x-a)^n}{n!}$$

bearing his name in 1715, and Stirling gave the Maclaurin expansion (the case $a = 0$) in the 1730's. Maclaurin's 1742 work *Treatise of Fluxions* contains those expansions. Incidentally, Taylor series appeared in earlier works of James Gregory and James Bernoulli – but that's another story.

15. (a) Gabriel Cramer. Credit could go to Leibniz (1683) or Maclaurin (ca. 1730), but we'll go with Cramer, who treated the $n \times n$ case in 1750. Kosinski (*Math. Magazine*, October 2001, 310-312) tells the whole story.

Bonus Question. (b) Someone else. It's only logical!

SUGGESTED READING

Carl B. Boyer and Uta C. Merzbach, *A History of Mathematics* (2nd edition), John Wiley & Sons, New York, NY, 1989.

David M. Burton, *The History of Mathematics: An Introduction* (6th edition), McGraw-Hill, New York, NY, 2006.

Howard M. Eves, *An Introduction to the History of Mathematics* (6th edition), Saunders/HBJ, New York, NY, 1990.

The following website has a wealth of information: begin here and follow the time line.
<http://lahabra.seniorhigh.net/pages/teachers/pages/math/timeline/MpreAndAncient.html>