

Nonlinear Model Reduction for Control

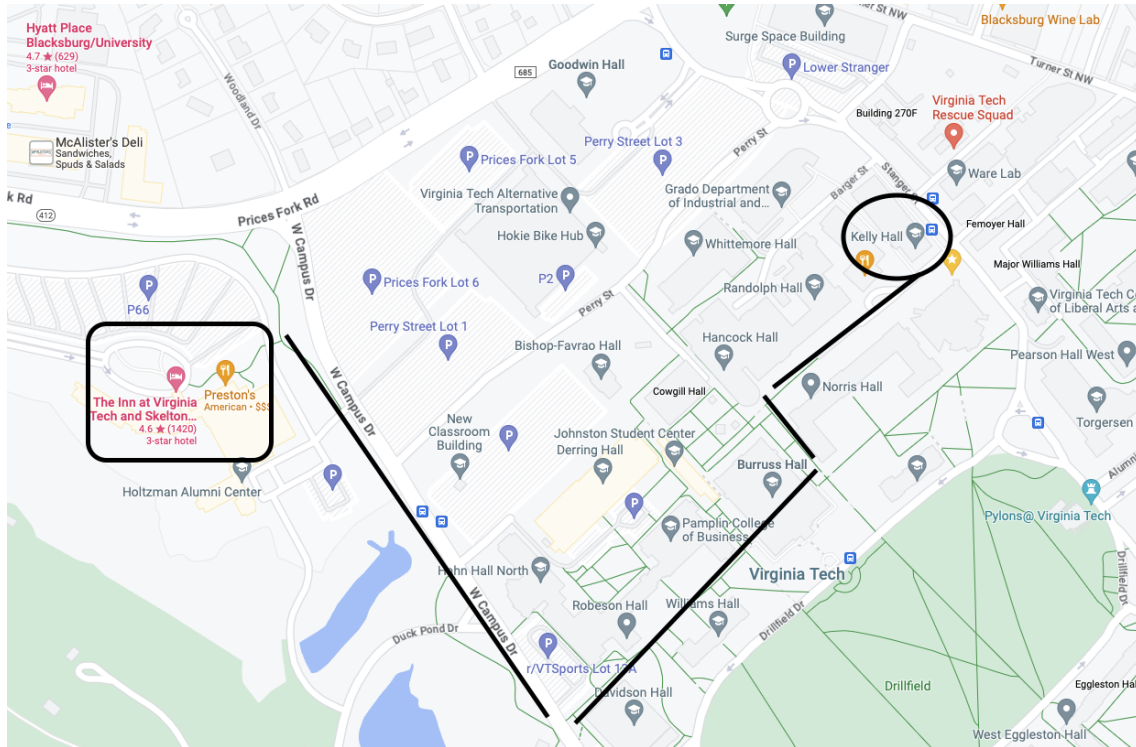
Workshop and Conference
22-26 May 2023
Virginia Tech, Blacksburg, VA



Location

All the talks (including the tutorial sessions) will take place in Kelly Hall 310.

The map below is a walking guide to avoid construction on Perry St.



Here is also a link, yet it may not know about ongoing construction: ([Google Maps](#))

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Nonlinear Model Reduction for Control

A tutorial-style workshop for junior researchers from 22-23 May 2023 followed by a conference on Nonlinear Model Reduction for Control 24-26 May 2023 will be held at Virginia Tech.

The two-day workshop is targeted at graduate students and postdocs, and will include tutorials on nonlinear balancing and control, proper orthogonal decomposition and other model reduction strategies, and computational methods for implementing these approaches.

Topics of the conference

The themes of interest to this conference are

- nonlinear balanced truncation,
- Hamilton-Jacobi-Bellman approaches to control and model reduction,
- interpolatory and data-driven methods for nonlinear systems,
- proper orthogonal decomposition for control, and
- tensor methods for scalable computation.

Conference Organizers

Jeff Borggaard, Virginia Tech (jborggaard@vt.edu)

Serkan Gugercin, Virginia Tech (gugercin@vt.edu)

Boris Kramer, UC San Diego (bmkramer@eng.ucsd.edu)

Financial Support

Participation costs for junior researchers was provided by the National Science Foundation under the program Dynamics, Control and System Diagnostics (CMMI-2130727).

Social Program

- Coffee breaks will be provided in the atrium outside Kelly 210.
- Lunch is on your own. There are a number of good places for lunch within a short walk of Kelly Hall. A detailed list of food options is available at [\(link\)](#).
- A social event is scheduled for Thursday evening from 6-8pm at the Eastern Divide Brewing Company. ([Google Maps](#)).

Tutorial Sessions

Monday, 22 May 2023		
	Speaker	Title
08:10-08:30	Registration and welcome	
08:30-09:30	Jacqueline Scherpen	Nonlinear balanced truncation
09:30-09:45	Coffee Break	
09:45-10:45	Jacqueline Scherpen	Nonlinear balanced truncation
10:45-11:00	Break	
11:00-12:00	Boris Kramer	Computation for nonlinear balancing
12:00-13:45	Lunch	
13:45-14:45	Boris Kramer	Computation for nonlinear balancing
14:45-15:00	Coffee Break	
15:00-16:00	Jeff Borggaard	Software for nonlinear balancing and control
16:00-16:15	Break	
16:15-17:15	Jeff Borggaard	Software for nonlinear balancing and control

Tuesday, 23 May 2023		
	Speaker	Title
08:30-09:30	Carmen Graessle	Control-oriented model reduction using POD
09:30-09:45	Coffee Break	
09:45-10:45	Carmen Graessle	Control-oriented model reduction using POD
10:45-11:00	Break	
11:00-12:00	Tobias Breiten	Nonlinear control and model reduction
12:00-13:45	Lunch	
13:45-14:45	Tobias Breiten	Nonlinear control and model reduction
14:45-15:00	Coffee Break	
15:00-16:00	Misha Kilmer	Tensor computations
16:00-16:15	Break	
16:15-17:15	Misha Kilmer	Tensor computations

Conference Program

Wednesday, 24 May 2023		
	Speaker	Title
08:30-09:00	Registration and welcome	
09:00-09:30	Elizabeth Qian	Balanced truncation and ensemble Kalman inversion
09:30-10:00	Josie Konig	Time-limited balanced truncation for data assimilation problems
10:00-10:30	Coffee Break	
10:30-11:15	John Singler	Incremental data compression for optimal control of time-dependent PDEs
11:15-11:45	Serkan Gugercin	Model reduction and data-driven modeling for nonlinear power networks
11:45-12:15	Christian Drischler	Applications of the reduced basis method in nuclear physics
12:15-14:00	Lunch	
14:00-14:30	Ion Victor Gosea	Data-driven balancing methods for classes of nonlinear systems
14:30-15:00	Jacquelin Scherpen	Extended differential balancing for nonlinear systems
15:00-15:30	Coffee Break	
15:30-16:00	Nicholas Corbin	Nonlinear balancing for quadratic bilinear systems
16:00-16:30	Linus Balicki	Balanced truncation for systems with polynomial outputs
16:30-17:00	Hamza Adjerid	Sum-of-squares approximations to energy functions

Thursday, 25 May 2023		
	Speaker	Title
08:30-09:00	Registration	
09:00-09:30	Jonas Nicodemus	On multi-objective model reduction of port-Hamiltonian systems
09:30-10:00	Harsh Sharma	Preserving Lagrangian structure in data-driven reduced-order modeling
10:00-10:30	Arjit Sarkar	Structure-preserving generalized balanced truncation for port-Hamiltonian systems
10:30-11:00	Coffee Break	
11:00-11:45	Jan Heiland	Low-dimensional linear parameter varying system approximations for nonlinear controller design
11:45-12:15	Steffen W.R. Werner	Context-aware learning for stabilizing dynamical systems from scarce data
12:15-14:00	Lunch	
14:00-14:30	Honghu Liu	Approximation of nonlinear optimal control problems in infinite dimension
14:30-15:00	Tobias Breiten	Approximations of operator Lyapunov equations with applications to nonlinear feedback control
15:00-15:30	Yichang Shen	Nonlinear model reduction for control of structures undergoing bifurcations
15:30-16:00	Coffee Break	
16:00-16:30	Mark Embree	Stability and transient dynamics for linearized reduced-order models
16:30-17:00	Alessandro Borghi	H2 optimal model order reduction on arbitrary domains
17:00-17:30	Petar Mlinarić	L2-optimal nonlinear parametric reduced-order modeling
18:00-20:00	Social event at Eastern Divide	

Friday, 26 May 2023		
	Speaker	Title
08:30-09:00	Registration	
09:00-09:30	Constantin Gahr	Data-driven non-intrusive reduced-order modeling for plasma turbulence via operator inference
09:30-10:00	Cankat Tilki	Wavelet-DMD in the context of extended-DMD
10:00-10:30	Sean Reiter	Interpolation-based \mathcal{H}_2 -optimal model reduction of systems with quadratic outputs
10:30-11:00	Coffee Break	
11:00-12:00	Presentation: NSF Dynamics, Controls and System Diagnostics (DCSD)	
12:00-12:15	Closing Remarks	

Conference Abstracts

Sum-of-squares approximations to energy functions

Hamza Adjerid

24 May
04:30pm
Kelly Hall

A limitation of polynomial approximations to past and future energy functions is ensuring positivity. This is particularly important when using the past energy function for control. Framing the approximation problem as a sum-of-squares ensures non-negativity. We investigate two approaches to finding sum-of-squares approximations. The first develops a sum-of-squares approximation that matches the first d components of a given polynomial that is locally positive definite, but at the expense of producing a higher-degree polynomial. The second is developed through a least-squares fit at sampled collocation points. Comparisons of these approaches using two examples are presented. A one-dimensional example with an analytic solution is used to highlight the quality of the collocation approach and a two-dimensional example compares the quality of the approximations when used in the control setting.

Balanced truncation for systems with polynomial outputs

Linus Balicki
Virginia Tech

24 May
04:00pm
Kelly Hall

Balanced Truncation aims to reduce the order of large-scale dynamical systems by truncating state-variables based on their relevance to the observability and controllability of the system. The process of determining the relevance of individual state-variables is connected to the energy functionals which are solutions to high-dimensional Hamilton-Jacobi equations. We consider linear dynamical systems whose output is represented by a polynomial function of the system's state and show that polynomial representations of energy functions can be computed by well-established solvers for large-scale linear systems of tensor structured equations. We further show that the newly obtained formulations for energy functionals can be used to derive balancing transformations allowing for balancing and reducing dynamical systems with polynomial outputs. Numerical results demonstrate that the derived approach is scalable and effective.

25 May
04:30pm
Kelly Hall

H2 optimal model order reduction on arbitrary domains

Alessandro Borghi
TU Berlin

The optimal H2 approximation has been widely used for the development of efficient model order reduction algorithms. Well known examples are IRKA and MIRIAM respectively adopted in continuous and discrete time state-space systems. These methods rely on the definition of a norm on a Hardy space from which the H2 optimal interpolation conditions follow. One of the main assumptions for IRKA and MIRIAM is that the transfer function of the full order model needs to be analytic on the right half complex plane and on the outside of the unit disk respectively. However, there can be cases in which the transfer function is analytic in domains that differ from the ones above. Hence, we propose a framework to derive first order interpolation conditions for H2 optimality in an arbitrary set. The theoretical background relies on conformal maps and generalizes Hardy spaces to functions that are analytic on specific domains. The objective is to eventually develop algorithms that can be used to find a reduced order transfer function that satisfies the H2 optimality conditions in the chosen set.

25 May
02:30pm
Kelly Hall

Approximations of operator Lyapunov equations with applications to nonlinear feedback control

Tobias Breiten
Technische Universität Berlin

Computing the Lyapunov function of a system plays a crucial role in optimal feedback control, for example when the policy iteration is used. In this talk we will focus on the Lyapunov function of a nonlinear autonomous finite-dimensional dynamical system that we will rewrite as an infinite-dimensional linear system using the Koopman operator. We show that this infinite-dimensional system has the structure of a weak-* continuous semigroup in a specially weighted Lp-space. This allows us to establish a connection between the solution of an operator Lyapunov equation and the desired Lyapunov function. It will be shown that the solution to this operator equation attains a rapid eigenvalue decay, which we will use to justify finite rank approximations with numerical methods. The usefulness for numerical computations will also be demonstrated with two short examples.

Nonlinear balancing for quadratic bilinear systems

Nicholas Corbin
UC San Diego

24 May
03:30pm
Kelly Hall

We present a numerical method to implement nonlinear balancing on quadratic-polynomial systems. Using Kronecker-product-based polynomial expansions of the nonlinear energy functions used for balancing, we convert the Hamilton-Jacobi-Bellman Partial Differential Equations (HJB PDEs) for the energy functions into a series of structured linear systems which are easily solvable for the polynomial coefficients of the energy functions.

Using a simple academic example for which the analytical energy functions are known, we demonstrate the method's ability to solve for an approximate polynomial representation of the energy function. Then we investigate the scalability of the approach by considering a model arising from the semi-discretization of a PDE, for which an analytical solution to the energy functions is computationally intractable.

Applications of the Reduced Basis Method in Nuclear Physics

Christian Drischler
Virginia Tech

25 May
11:45am
Kelly Hall

Nuclear systems are notoriously complex. But typically, our theoretical modeling of nuclear phenomena contains superfluous information for quantities of interest. The reduced basis method (RBM) has facilitated applications of Bayesian statistical methods for rigorous uncertainty quantification of microscopic nuclear models and much more. Some of these applications were deemed impossible due to their computational demands. Nevertheless, the nuclear physics community has barely scratched the surface of model reduction methods and is eager to learn from the experts in the SIAM community. In this talk, I will highlight recent RBM-driven approaches to address key questions in nuclear science, such as how do properties of nuclei such as Gold emerge from fundamental principles of the strong interaction?

Stability and transient dynamics for linearized reduced-order models

25 May
04:00pm
Kelly Hall

Mark Embree
Virginia Tech

Projection methods provide an appealing, intuitive strategy for reducing linear dynamical systems. In general, however, they need not preserve the stability of the original system. How many unstable modes can there be? Applying results from Russell Carden and Anne Greenbaum, we will address this question for Galerkin and Petrov-Galerkin methods. What can be learned from these unstable modes? When the system arises from linearization of a nonlinear dynamical system, unstable modes in the reduced model can provide insight into transient dynamics and potential nonlinear instability.

Data-driven non-intrusive reduced-order modeling for plasma turbulence via operator inference

26 May
09:00am
Kelly Hall

Constantin Gahr
Max-Planck-Institute for Plasma Physics

Turbulence simulations play a crucial role in the plasma physics community as they give insight into the underlying nonlinear dynamics. However, these simulations are computationally expensive. Reduced-order models provide a computationally cheaper alternative to the high-fidelity model exploiting the fact that in most physics and engineering problems, the dominant dynamics live on low-dimensional manifolds. We focus on the Hasegawa-Wakatani equations, a plasma model describing two-dimensional drift-wave turbulence, and approximate it with a reduced order model learned via Operator Inference. Operator Inference is a data-driven non-intrusive model reduction method that learns low-dimensional reduced models with polynomial nonlinearities from trajectories of high-dimensional high-fidelity simulations. In addition, it can handle arbitrary nonlinearities by employing lifting transformations that map the given states into states with polynomial nonlinearities. In the present work, we perform one of the first systematic reduced-order modeling studies in plasma physics to ascertain whether Operator Inference can provide accurate and predictive reduced models for the Hasagawa-Wakatani system.

Data-driven balancing methods for classes of nonlinear systems

Ion Victor Gosea¹ with Igor Pontes Duff, Serkan Gugercin² and Christopher Beattie²

¹Max Planck Institute, Magdeburg, Germany; ²Virginia Tech

24 May
02:00pm
Kelly Hall

We propose a non-intrusive data-driven formulation of balanced truncation (BT) for bilinear systems by extending the recently-proposed approach in Gosea, Gugercin, and Beattie 2022 known as quadrature-based BT (QuadBT) to the bilinear setting. QuadBT recasts the classic (intrusive) BT method for linear time-invariant systems as a data-driven method, solely requiring samples of either transfer function values (in the frequency domain) or impulse response (in the time domain). In this work, we extend the range of applicability of QuadBT to the class of bilinear time-invariant systems, an important sub-class of weakly-nonlinear dynamical systems. We emphasize mainly the case of time-domain data, which requires evaluations of the (generalized) impulse responses, and also of their derivatives. We present numerical test cases for which such data can be generated/inferred through direct numerical simulation. If time permits, we will also show extensions to modeling quadratic systems using the data-driven balancing approach.

Model reduction and data-driven modeling for nonlinear power networks

Serkan Gugercin
Virginia Tech

24 May
11:15am
Kelly Hall

Power networks are complex and large-scale systems in operation. Simulating the corresponding models are computationally expensive and demand unmanageable levels of storage in dynamic simulation or trajectory sensitivity analysis. Therefore, model reduction of power network dynamics is essential for real time applications.

We first present a projection-based structure-preserving model reduction framework for nonlinear power grid networks. After converting the original nonlinear system to an equivalent quadratic nonlinear model, we employ a wide range of input-independent model reduction techniques that also retain critical nonlinear structural features of the original system. Next we consider complementary data-driven approaches to power network modeling where access to internal dynamics are not available. Using the time-domain state-snapshot data, we construct structured surrogates for nonlinear swing equations.

Plenary: Low-dimensional linear parameter varying system approximations for nonlinear controller design

25 May
11:00am
Kelly Hall

Jan Heiland

Max Planck Institute for Dynamics of Complex Technical Systems

The computer-aided control system design for nonlinear large-dimensional systems is a tremendous task as it has to cope both with model complexities due to nonlinear dynamics and with computational challenges that are caused by the system's size. While linear algebra packages and algorithms for solving, say, Riccati equations can handle the large system sizes, the theory for linear parameter varying (LPV) systems has been successfully employed for nonlinear controller design. Within this talk, we want to lay out how to make the LPV theory accessible for numerical controller design for nonlinear large-scale systems and discuss controller design techniques and numerical examples of incompressible Navier-Stokes flow.

The core idea is to use approximations to provide low-dimensional LPV surrogate models for nonlinear input-affine systems

$$\dot{x}(t) = f(x(t)) + Bu(t), \quad y(t) = Cx(t),$$

where for time $t > 0$, $x(t) \in R^n$ denotes the state, $u(t) \in R^p$ and $y(t) \in R^q$ denote the input and output, where $f: R^n \rightarrow R^n$ is a possibly nonlinear function, and where B and C are linear input and output operators.

Based on state-dependent coefficient representations $f(x) = A(x)x$, which always exist under mild conditions on f and which are naturally given for quadratic systems like the Navier-Stokes equations, we derive low-dimensional LPV approximations for the nonlinear control system using standard model order reduction techniques.

For systems in LPV form, there exist well established numerical techniques for controller design. We briefly introduce the main ideas and discuss how these apply to large scale systems (note that the LPV approximation only reduces the system's structure but not the state space dimension).

In a numerical example of a two-dimensional flow control problem, we illustrate and evaluate the major steps of the approximation and the controller design.

Time-limited balanced truncation for data assimilation problems

Josie König
University of Potsdam

24 May
09:30am
Kelly Hall

Balanced truncation is a well-established model order reduction concept in system theory that has been applied to a variety of problems. Recently, a connection between linear Gaussian Bayesian inference problems and the system theoretic concept of balanced truncation was drawn for the first time. Although this connection is new, the application of balanced truncation to data assimilation is not a novel concept: It has already been used in four-dimensional variational data assimilation (4D-Var) in its discrete formulation. In my work, the link between system theory and data assimilation is further strengthened by discussing the application of balanced truncation to standard linear Gaussian Bayesian inference, and, in particular, the 4D-Var method. Similarities between both data assimilation problems allow a discussion of established methods as well as a generalisation of the state-of-the-art approach to arbitrary prior covariances as reachability Gramians. Furthermore, we propose an enhanced approach using time-limited balanced truncation that allows to balance Bayesian inference for unstable systems and in addition improves the numerical results for short observation periods.

Approximation of nonlinear optimal control problems in infinite dimension

Honghu Liu
Virginia Tech

25 May
02:00pm
Kelly Hall

This presentation is concerned with the optimal control of nonlinear evolution equations in Hilbert spaces. We will present checkable conditions that guarantee the convergence of the value functions associated with the optimal control problem of the corresponding spectral Galerkin approximations. We will show that this convergence result holds for a broad class of nonlinear evolution equations and cost functionals as well as for a broad class of nonlinear control strategies. Applications in various contexts will also be illustrated that include semilinear heat equations posed on a general compact manifold without boundary, population dynamics governed by a Kolmogorov-Petrovsky-Piskunov equation with a harvesting term and space-dependent coefficients, and equations involving time-delay terms.

25 May
05:00pm
Kelly Hall

L2-optimal nonlinear parametric reduced-order modeling

Petar Mlinarić
Virginia Tech

We consider L2-optimal reduced-order modeling for nonlinear parametric stationary systems, based on parameter-separable forms. We present results for linear reduced-order models and extensions to classes of nonlinear systems. Based on this, we propose a gradient-based optimization algorithm for finding locally optimal reduced-order models. Furthermore, we present the necessary optimality conditions in the interpolation form, related to interpolatory H2-optimality conditions. Finally, we demonstrate the results on a number of numerical examples.

25 May
09:00am
Kelly Hall

On multi-objective model reduction of port-Hamiltonian systems

Jonas Nicodemus
University of Stuttgart

Conventional structure-preserving model order reduction (MOR) methods for port-Hamiltonian (pH) systems focus on approximating the input-output dynamics by (approximately) minimizing classical system norms, such as the Hardy norm. Nevertheless, the definition of a linear pH system consists of two objects: the linear input-output dynamics and a nonlinear energy function that is typically called the Hamiltonian. If we thus measure the approximation quality only with respect to the input-output dynamics, then the approximation of the Hamiltonian is not reflected at all. This is particularly relevant since recent results [Breiten and Unger, Automatica 142, 2022] demonstrate that modifying the Hamiltonian of the full order model (FOM) may yield better reduced order models (ROMs), at least if only the input-output dynamics are analyzed. In this talk, we make a first step towards the dual-objective minimization problem of optimal approximation of the input-output dynamics and the Hamiltonian by noticing that in the pH ROM, we can modify the nonlinear Hamiltonian without changing the system dynamics. Thus, for a given pH ROM, we can search for the reduced Hamiltonian that best approximates the FOM Hamiltonian. We prove that the resulting optimization problem is strictly convex and uniquely solvable. Moreover, we propose a numerical algorithm to solve the minimization problem and demonstrate its applicability with numerical examples.

Balanced truncation and ensemble Kalman inversion

Elizabeth Qian

Georgia Tech

24 May
09:00am
Kelly Hall

The Ensemble Kalman filter is a classical method for solving data assimilation problems in which a stream of incoming data is used to update the underlying state of a model sequentially in time. The Ensemble Kalman Inversion (EKI) approach re-interprets the ensemble Kalman filter as a method for solving more general inverse problems. For high-dimensional smoothing problems (where the state to be inferred is the unknown initial condition and the indirect measurements are taken at later times) a high-dimensional ODE must be solved for each particle at each iteration of the EKI approach. Because the EKI error scales at the Monte Carlo rate this cost can be prohibitive. Balanced truncation for Bayesian inference (BTBI) is a recently introduced model reduction method for linear dynamical systems tailored to the Bayesian smoothing problem. This talk will introduce EKI and BTBI and describe how their union leads to efficient solution of the smoothing problem.

Interpolation-based \mathcal{H}_2 -optimal model reduction of systems with quadratic outputs

Sean Reiter

Virginia Tech

26 May
10:00am
Kelly Hall

In this work, we investigate the \mathcal{H}_2 model reduction problem for linear dynamical systems with quadratic outputs (LQO) in an interpolation framework. Our primary contributions are twofold: First, we derive interpolation-based first-order optimality conditions for \mathcal{H}_2 model reduction. These constraints amount to rational interpolation of a univariate and multivariate transfer function of the underlying model; these describe the linear and quadratic parts of the output, respectively. Secondly, we show how this interpolation-based approach can be realized non-intrusively and entirely from sampling data of the two transfer functions of the full-order model. In either case, we adapt the iterative rational Krylov algorithm as an efficacious approach for computing reduced-order LQO models that satisfy the necessary optimality conditions. Numerical experiments illustrate the efficiency and effectiveness of the proposed interpolation framework.

Structure-preserving generalized balanced truncation for port-Hamiltonian systems

25 May
10:00am
Kelly Hall

Arjit Sarkar

University of Groningen

In this work, we introduce the notions of generalized controllability and observability functions for stable continuous-time nonlinear systems. We propose a balanced realization for nonlinear port-Hamiltonian systems in which the generalized energy functions are balanced and at the same time the Hamiltonian of the corresponding system is in diagonal form. The reduced order model obtained by truncation of states in the balanced coordinates preserves the port-Hamiltonian structure. For a special case of nonlinear port-Hamiltonian systems with quadratic Hamiltonian, the approach of generalized balanced truncation boils down to a feasibility problem of matrix inequalities. We illustrate the results with an example of a mass–spring–damper system with Coulomb friction.

Extended differential balancing based model reduction for structure preservation

24 May
02:30pm
Kelly Hall

Jacqueline Scherpen

In this talk, we develop extended balancing techniques for linear and nonlinear systems with the aim to have methods for control that preserve additional structure, such as a port-Hamiltonian system structure. The nonlinear methods are developed in the contraction framework. We use the dissipativity framework to introduce the concept of the extended differential observability and the inverse of the extended differential controllability Gramians, and we show the relation with generalised balancing. We then use these for model reduction to get a smaller a priori error bound in comparison with other types of balancing. We show the utilisation of these methods for structure preserving model reduction of port-Hamiltonian systems.

Preserving Lagrangian structure in data-driven reduced-order modeling

25 May
09:30am
Kelly Hall

Harsh Sharma

UC San Diego

We present a nonintrusive physics-preserving method to learn reduced-order models (ROMs) of Lagrangian systems, which includes nonlinear wave equations. Existing intrusive projection-based model reduction approaches construct structure-preserving Lagrangian ROMs by projecting the Euler-Lagrange equations of the full-order model (FOM) onto a linear subspace. This Galerkin projection step requires complete knowledge about the Lagrangian operators in the FOM and full access to manipulate the computer code. In contrast, the proposed Lagrangian operator inference approach embeds the mechanics into the operator inference framework

to develop a data-driven model reduction method that preserves the underlying Lagrangian structure. The method does not require access to FOM operators or computer code. The numerical results demonstrate Lagrangian operator inference on an Euler-Bernoulli beam model, the sine-Gordon (nonlinear) wave equation, and a large-scale discretization of a soft robot fishtail with 779,232 degrees of freedom. For the high-dimensional soft-robotic fishtail model, the learned Lagrangian ROMs track the change in system energy accurately in the presence of dissipation and time-dependent control input and work well even for unknown control inputs. Moreover, our method achieves significant reduction in state dimension, which makes the learned ROMs ideal for real-time control and state estimation.

Nonlinear model reduction for control of structures undergoing bifurcations

Yichang Shen
Imperial College London

25 May
03:00pm
Kelly Hall

Control-based continuation is a means to apply the principles of numerical continuation to a physical system and perform bifurcation analysis directly during experiments. To facilitate the design of the feedback controller and improve its performance, it is advantageous to use a reduced-order model capturing the dynamics of the tested system. In this talk, we investigate the control of a nonlinear cantilever beam model exhibiting saddle-node and Neimark Sacker bifurcations. We employ a reduced-order model obtained by normal form reduction to generate the prediction used in model predictive control in order to track periodic reference signals. This approach is compared to a linear model predictive control approach based on an approximation of the Koopman operator using EDMD.

Plenary: Incremental data compression for optimal control of time-dependent PDEs

John Singler

24 May
10:30am
Kelly Hall

We propose an incremental proper orthogonal decomposition (iPOD) gradient descent method to address the data storage issue in solving optimal control problems for time-dependent PDEs. We review the iPOD, which is an incremental singular value decomposition algorithm with a weighted inner product, and give an improved error analysis of the method. We consider two example problems and show that the proper choice of Hilbert space for iPOD gradient descent yields well-controlled errors and convergence. We present numerical results that show computer memory can be greatly saved without compromising convergence of the gradient descent method or the accuracy of the optimal solution.

26 May
09:30am
Kelly Hall

Wavelet-DMD in the context of extended-DMD

Cankat Tilki
Virginia Tech

Dynamic Mode Decomposition and its variants have been successfully used in data driven modeling of dynamical systems in diverse applications. DMD has been generalized so that it admits external forcing and outputs, giving rise to the input-output (ioDMD). DMD and ioDMD work with full state measurements. However in practice one does not always have access to full measurements. This leads to versions of DMD, such as delay-DMD, that use partial state measurements rather than full ones.

One of the recent techniques that works with partial state information in a DMD setting is Wavelet-DMD (WDMD). Main idea behind this method is to create observables based on the stationary wavelet coefficients using the available measurements and apply ioDMD with these observables. Specifically, WDMD uses maximal overlap discrete wavelet transform to generate an auxiliary state variable from partial measurements, and then uses ioDMD to analyze dynamics with respect to this auxiliary variable and creates a data-driven model for the underlying dynamics. In this talk, we demonstrate that WDMD can be represented formally in the context of Extended DMD with a specific choice of observables. Thus we establish the connection between WDMD and the extensions of DMD. We also provide numerical examples to illustrate the theory.

25 May
11:45am
Kelly Hall

Context-aware learning for stabilizing dynamical systems from scarce data

Steffen W.R. Werner
Courant Institute of Mathematical Sciences, New York University

Stabilizing dynamical systems in science and engineering is challenging, especially in edge cases and limit states where typically little data are available. Leveraging ideas from model reduction, we propose a data-driven approach that guarantees finding stabilizing controllers from as few data samples as the dimension of the unstable dynamics, which typically is orders of magnitude lower than the state dimension of the system. The key is learning stabilizing controllers directly from data without learning models of the system dynamics, which would require larger numbers of data points. Numerical experiments with chemical reactors and fluid dynamics behind obstacles demonstrate that the proposed approach stabilizes systems after observing fewer than five data samples even though the dimension of states is orders of magnitude higher.

List of Participants

Mike Ackermann	Virginia Tech
Iman Adibnazari	University of California San Diego
Hamza Adjerid	Virginia Tech
Camilo Arenas	Virginia Tech
Linus Balicki	Virginia Tech
Jeff Borggaard	Virginia Tech
Ali Bouland	Virginia Tech
Alessandro Borghi	TU Berlin
Tobias Breiten	Technische Universität Berlin
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