Numerical Linear Algebra

Practical solution of linear algebra problems
→ efficient algorithms
→ properties of (numerical) solutions

NLA is of immense practical importance.

At the heart of most large-scale simulations, optimizations, analysis of systems, ... we find (very large) linear algebra problems.

(Often NLA part takes almost all the time)

Why NLA central to simulation, optimization ?

Most important linear algebra problems:

- system of linear equations
- overdetermined system of linear equations
- underdetermined system of linear eq.s (with appropr. constraints / conditions)
- eigenvalue problems
- approximation of matrix functions

Why NLA central ?

1) For more than a few var.s, linear systems are the only ones we can solve directly (finite number of calculations)

2) Many/most nonlinear problems (most prob.s ~~the~~ science & engineering) can be approximated (locally) by linear problems →
Solve nonlinear problems by sequence of linear problems.

Nonlinear (system of) part. diff. equations → ~~lin~~ system linear PDEs. Discretization gives (large) system of linear equations.

Alternatively, we can discretise first and solve system nonlinear algebraic equations (sequence of linear problems).

3) Best approx. of (unknown/known) function in subspace →

orthogonal projection

Very general principle with many applications!

(related to solving overdetermined system of equations)

4) Problems of vibrations, stability, resonance are related to eigenvalue problems.

(local) linearization gives standard linear or generalized eigenvalue problem.

But more generally we can also consider nonlinear or polynomial eigenvalue problems.

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Eigenvalues and vectors also play an important role in the solution of very large systems of ODEs

We will discuss much of this later, including applications.

~~Note that not only does NLA play an important role in solving (numerically) some linear algebra problem after resulting from some approach to solve the original problem, but the~~

Apart from the role NLA plays in solving some LA problem resulting from algorithm to solve original problem, linear algebra techniques/principles play fundamental role in deriving solutions ~~for~~ (algorithms)

Overview

Book:

I Linear systems of equations

Chapters 1+2

- direct solution for various types of problems
- algorithms and cost
- accuracy, sensitivity, stability
- applications

II Overdetermined and underdetermined problems

Chapters 3+4 + notes

- Overdet. problems:
  - x principles, least squares
  - x solution, general principles
  - x algorithms and cost
  - x accuracy, sensitivity, stability
  - x applications
  - x singular value decomposition (SVD)
- Underdetermined problems:
  - x principles, min. norm solutions
  - x other types of solution

III Eigenvalue and singular value problems

Chapters 5+6

- definitions / basics
- algorithms and analysis
- algorithms for a few eigenpairs of large (sparse) matrices
- algorithms for the SVD
- accuracy, sensitivity, stability
- generalized & polynomial eig. value problems

(Eigenvalue problems continued)

- applications

Chapter 7 + notes

IV Iterative methods for linear systems

- fixed point iterations
- Krylov subspace methods

notes

V Advanced topics

- nonlinear equations and optimization
- model reduction